D (A)

1 PODMÍNKY: a) $\frac{b^2-1}{b+1}$ $b \neq -1$; b) $\frac{s-9}{s^2-9} = \frac{s-9}{(s-3)(s+3)}$ $s \neq \pm 3$; c) $\frac{2}{4a^2-4a+1} = \frac{2}{(2a-1)^2}$ $2a-1 \neq 0$, $2a \neq 1$, $a \neq 1,2$

II KRÁCENÍ A SOUČIN

1. a) $\frac{a^4}{a^2} \cdot \frac{a^2 \cdot a^2}{a^2} = a^2$ (nebo $\frac{a^4}{a^2} = a^{4-2} = a^2$) $a \neq 0$; b) $\frac{15xy}{6x^2y^0} = \frac{5}{2x}$ $x \neq 0, y \neq 0$ c) $\frac{3v}{2v^2+v} = \frac{3v}{v(2v+1)}$ $v \neq 0$, $2v+1 \neq 0$, $2v \neq -1$, $v \neq -0,5$

d) $\frac{2u-1}{4u^2-4u+1} = \frac{2u-1}{(2u-1)^2}$ $2u-1 \neq 0$, $2u \neq 1$, $u \neq 0,5$

2. a) $\frac{t^3}{6} \cdot \frac{15}{t^5} = \frac{5t^2}{2}$ (nebo $\frac{5}{2}t^2$; $2{,}5t^2$) $t \neq 0$ b) $\frac{m}{m-1} \cdot \frac{(m-1)^2}{m^2} = \frac{m-1}{m}$ $m \neq 0$, $m \neq 1$

3. a) $\frac{15a^2+10ab}{15ab+10b^2} = \frac{5a(3a+2b)}{5b(3a+2b)} = \frac{a}{b}$ $b \neq 0$, $3a \neq -2b$, $a \neq -\frac{2}{3}b$ b) $\frac{3x^4-6x^2y+3y^2}{6x^2-6y} = \frac{3(x^4-2x^2y+y^2)}{6(x^2-y)} =$

$= \frac{(x^2-y)^2}{2(x^2-y)} = \frac{1}{2(x^2-y)}$ $x^2 \neq y$

4. a) $\frac{16u^2-16u+4}{3u+3} : \frac{4(4u^2-4u+1)}{3(u+1)} = \frac{4(2u-1)^2}{3(u+1)}$

$\frac{16u^2-16u+4}{3u+3} \cdot \frac{1+u}{16u^2-4} = \frac{4(4u^2-4u+1)}{3(u+1)} \cdot \frac{(1+u)}{4(4u^2-1)} = \frac{4(2u-1)^2}{3} \cdot \frac{1}{4(2u-1)(2u+1)} = \frac{4}{3(2u+1)}$

$u \neq -1$; $u \neq \pm 0,5$

b) $\frac{2x+8}{x^3} \cdot \frac{x^2-xy}{x+4} = \frac{2 \cdot (x+4)}{x^3} \cdot \frac{x(x-y)}{(x+4)} = \frac{2(x-y)}{x^2}$ $x \neq 0$; $x \neq -4$

5. a) $\frac{s^2-10s+25}{-6s+30} = \frac{(s-5)^2}{-6(s-5)} = \frac{s-5}{-6}$ (další "komutická" úprava) $= \frac{5-s}{6}$ $s \neq 5$

b) $\frac{ay^2-ax^2}{(a+b)^2} \cdot \frac{3a+3b}{ax^2-2axy+ay^2} = \frac{a(y^2-x^2)}{(a+b)^2} \cdot \frac{3(a+b)}{a(x^2-2xy+y^2)} = \frac{-1(x^2-y^2)}{(a+b)} \cdot \frac{3}{(x-y)^2} =$

$= \frac{-(x+y)(x+y)}{(a+b)} \cdot \frac{3}{(x-y)^2} = \frac{-3x-3y}{(a+b)(x-y)}$ $a \neq -b$, $x \neq y$

III SOUČET

6. a) $\frac{3x-2}{x-1} + \frac{1-2x}{x-1} = \frac{3x-2+1-2x}{(x-1)} = \frac{x-1}{x-1} = 1$ $x \neq 1$; b) $\frac{1}{a} + \frac{2x}{b} = \frac{1b+2x \cdot a}{ab} = \frac{b+2xa}{ab}$ $a \neq 0$; $b \neq 0$

b) $\frac{(r+1)}{r} - 1 = \frac{(r+1)-1 \cdot r}{r} = \frac{r+1-r}{r} = \frac{1}{r}$ $r \neq 0$;

7. a) $\frac{2x}{x+y} - \frac{1}{x-y} = \frac{2x(x-y)-1 \cdot (x+y)}{(x+y)(x-y)} = \frac{2x^2-2xy-x-y}{x^2-y^2}$ $x \neq \pm y$

b) $\frac{3x}{3x-1} - \frac{2}{(3x-1)^2} = \frac{3x(3x-1)-2}{(3x-1)^2} = \frac{9x^2-3x-2}{(3x-1)^2}$ $3x \neq 1$, $x \neq \frac{1}{3}$

8. $\frac{2v}{6v+6} - \frac{3}{9v^2+18v+9} = \frac{2v}{6(v+1)} - \frac{3}{9(v^2+2v+1)} = \frac{2v}{3 \cdot 2(v+1)} - \frac{3}{3 \cdot 3(v+1)^2} = \frac{2v}{3(v+1)} - \frac{1}{3(v+1)^2} =$

$= \frac{v(v+1)-1}{3(v+1)^2} = \frac{v^2+v-1}{3(v+1)^2}$ $v \neq -1$

IV PODÍL

9. a) $\frac{2}{v^2} : \frac{4}{v} = \frac{2}{v^2} \cdot \frac{v}{4_2} = \frac{1}{2v}$ $v \neq 0$ b) $\frac{3x}{x+y} : \frac{(x+y)^2}{x1} = \frac{3x}{(x+y)(x+y)} \cdot \frac{x1}{7} = \frac{x(x+y)}{7}$

$x \neq y$

10. a) $\frac{u^2-4}{u-2} : \frac{(u-2)^2}{u-2} = \frac{(u-2)(u+2)}{(u-2)} : \frac{(u-2)}{1} = \frac{u+2}{1} \cdot \frac{1}{u-2} = \frac{u+2}{u-2}$ $u \neq 2$

LZE KRÁTIT V JEDNOM ZLOMKU
A NE V DO CELÉ

u) $\frac{ax-ay}{(x+y)^2} : \frac{a(x-y)^2}{3x+3y} = \frac{a(x^2-y^2)}{(x+y)^2} \cdot \frac{3(x+y)}{a(x-y)^2} = \frac{(x-y)(x+y)}{(x+y)} \cdot \frac{3}{(x-y)^2} = \frac{3}{x-y}$

$x \neq -y$
$x \neq y$

V. SLOŽENÝ

12. a) $\frac{\frac{2ab}{3xy}}{\frac{2ax}{3by}} = \frac{2ab}{3xy} : \frac{2ax}{3by} = \frac{2ab}{3xy} \cdot \frac{3by}{2ax} = \frac{b \cdot b}{x \cdot x} = \frac{b^2}{x^2}$ $x \neq 0$, $y \neq 0$, $b \neq 0$, $a \neq 0$

b) $\frac{\frac{x^2-1}{2x}}{\frac{x+1}{8x^2}} = \frac{x^2-1}{2x} : \frac{x+1}{8x^2} = \frac{(x-1)(x+1)}{2x} \cdot \frac{8x^2}{(x+1)} = 4x \cdot (x-1)$ $x \neq 0$, $x \neq -1$

13. a) $\frac{\frac{2r+2s}{3r-3s}}{\frac{6r+6s}{r^2-rs}} = \frac{2(r+s)}{3(r-s)} : \frac{6(r+s)}{r(r-s)} = \frac{2(r+s)}{3(r-s)} \cdot \frac{r(r-s)}{6(r+s)} = \frac{r}{9}$ $r \neq \pm s$, $r \neq 0$

b) $\frac{\frac{a+b}{a-b}}{a^2+2ab+b^2} = \frac{a+b}{a-b} \cdot \frac{1}{(a+b)^2} = \frac{1}{(a-b)(a+b)} = \frac{1}{a^2-b^2}$ $a \neq \pm b$

VI PŘÍJMACKOVÉ PŘÍKLADY

a) $\left(\frac{1}{1-a^{-1}}\right) : \left(a - \frac{1-2a^2}{1-a} + 1\right) = \left(\frac{1-1+a}{1-a}\right) : \left(\frac{a(1-a)-(1-2a^2)+1\cdot(1-a)}{1-a}\right) =$

$= \frac{a}{1-a} : \frac{a-a^2-1+2a^2+1-a}{1-a} =$

$= \frac{a}{1-a} \cdot \frac{1-a}{a^2} = \frac{1}{a}$ $a \neq 1$, $a \neq 0$

b) $\frac{1-\frac{2b}{a}+\frac{b^2}{a^2}}{\frac{a-b}{a^3}} = \frac{\frac{a^2-2ab+b^2}{a^2}}{\frac{a-b}{a^3}} = \frac{(a-b)^2}{a^2} : \frac{a-b}{a^3} = \frac{(a-b)^2}{a^2} \cdot \frac{a^3}{(a-b)} = a(a-b)$

$a \neq 0$; $a \neq b$

c) $\left(\frac{n+2}{n-2} + \frac{2}{n^2-4} - \frac{n}{n+2}\right) \cdot \left(2 - \frac{n+4}{n+1}\right) =$

$= \frac{(n+2)(n+2)+2-n(n-2)}{(n-2)(n+2)} \cdot \frac{2(n+1)-(n+4)}{n+1} =$

$= \frac{n^2+4n+4+2-n^2+2n}{(n-2)(n+2)} \cdot \frac{2n+2-n-4}{n+1} = \frac{6n+6}{(n-2)(n+2)} \cdot \frac{n-2}{n+1} =$

$= \frac{6(n+1)}{(n+2)} \cdot \frac{1}{(n+1)} = \frac{6}{n+2}$ $n \neq \pm 2$, $n \neq 1$

(5)

I URČOVÁNÍ PODMÍNEK

1. a) $\frac{a^2-b^2}{b-2}$ $b+2$; b) $\frac{m-8}{m^2-m} = \frac{m-8}{m(m-1)}$ $m \neq 0$, $m \neq 1$ c) $\frac{2}{16c^2-16c+1} = \frac{2}{4-1}$

II KRÁCENÍ, SOUČIN

2. a) $\frac{r^3}{r^2} = r$ $r \neq 0$; b) $\frac{3k^2l}{6kl^2} = \frac{k}{2l}$ $k \neq 0$, $l \neq 0$ c) $\frac{x-2}{5x-10} \cdot \frac{x-2}{5(x-2)}$ $x \neq 2$ d) $\frac{a+b}{a^2+2ab+b^2} = \frac{a+b}{(a+b)^2}$ $a \neq -b$

3. a) $\frac{6x+2xy}{2a^2+4a} = \frac{6x+2xy}{2a(a+2)}$ $a \neq 0$, $a \neq -2$ b) $\frac{8v^2-8v+2}{8v-1}$ $v = 0{,}5$

4. a) $\frac{m^2-mn}{m^2+mn} \cdot \frac{m^2n+mn^2}{mn} = \frac{m(m-n)}{m(m+n)} \cdot \frac{mn(m+n)}{mn} = m-n$

b) $\frac{r^2-9}{r+1} \cdot \frac{r^2-1}{r-3} = \frac{(r-3)(r+3)}{(r+1)} \cdot \frac{(r-1)(r+1)}{r-3} = (r+3)(r-1)$ $r \neq -1$, $r \neq 3$

5. a) $\frac{9z^3-27vz}{3vz+z^2} = \frac{9z(z^2-3v)}{z(3v+z)} = \frac{9(z^2-3v)}{(3v+z)}$ $z \neq 0$, $-3v \neq z$ b) $\frac{v^2-9}{r+1} \cdot \frac{r^2-1}{r-3} = \frac{(r-3)(r+3)}{r+1} \cdot \frac{(r+1)(r-1)}{(r-3)} =$ $= (r+3)(r-1)$ $r \neq -1$, $r = 3$

III SOUČET LV

6. a) $\frac{a-3}{a+2} + \frac{3-a}{a+2} = \frac{a-3+3-a}{a+2} = \frac{0}{a+2} = 0$ $a \neq -2$ b) $\frac{3v}{k} - \frac{2}{k^2} = \frac{3vk-2}{k^2}$ $k \neq 0$;

c) $\frac{2l}{l-2} + 1 = \frac{2l+1 \cdot (l-2)}{l-2} = \frac{2l+l-2}{l-2} = \frac{3l-2}{l-2}$ $l \neq 2$

7. a) $\frac{a}{2a+b} + \frac{b}{(2a+b)^2} = \frac{a \cdot (2a+b)+b}{(2a+b)^2} = \frac{2a^2+ab+b}{(2a+b)^2}$ $2a \neq -b$ b) $\frac{2y+x}{x-y} - \frac{x+y}{2y-2x} =$

b) $\frac{2y+x}{x-y} - \frac{x+y}{2y-2x} = \frac{2y+x}{x-y} - \frac{x+y}{(-2)(x-y)} = \frac{-2(2y+x)-(x+y)}{(-2)(x-y)} = \frac{-4y-2x-x-y}{(-2)(x-y)} =$

$= \frac{-5y-3x}{(-2)(x-y)} = \frac{(-1)(5y+x)}{(-2)(x-y)} = \frac{x+5y}{2(x-y)}$ $x \neq y$

8. a) $\frac{3}{y+1} + \frac{3-3y^2}{y^3-y} = \frac{3}{y+1} + \frac{3(1-y^2)}{y(y^2-1)} = \frac{3}{y+1} + \frac{3(1-y^2)}{y(y-1)(y+1)}$

$= \frac{3}{y+1} + \frac{(-3)(y^2-1)}{y(y^2-1)} = \frac{3}{y+1} - \frac{3}{y} = \frac{3y-3(y+1)}{y(y+1)} = \frac{3y-3y-3}{y(y+1)} =$

$= \frac{-3}{y(y+1)}$ $y \neq -1$, $y \neq +1$, $y = 0$

b) $\frac{3}{a-b} + \frac{2(b-a)}{a^2-2ab+b^2} = \frac{3}{a-b} + \frac{(-2)(a-b)}{(a-b)^2} = \frac{3-2}{(a-b)} = \frac{1}{a-b}$ $a \neq b$

IV PODÍL LV

9. viz skup. A

10. $\frac{(x+y)^2-4}{x^2-y^2} : \frac{x+y}{x-y} = \frac{x^2+2xy+y^2-4}{(x-y)(x+y)} \cdot \frac{(x-y)}{(x+y)} = \frac{[(x+y)-2][(x+y)+2]}{(x+y)^2}(x^2+2xy+y^2-4)$

uvnitř závorky!

$x \neq \pm y$

(B)

10d) $\frac{r+3}{r-3} : \frac{r^2+3r}{2r^2-18} = \frac{r+3}{r-3} \cdot \frac{2(r^2-9)}{r(r+3)} = \frac{r+3}{r-3} \cdot \frac{2(r-3)(r+3)}{r(r+3)} = \frac{2(r+3)}{r}$ $r \neq 3$, $r \neq -3$, $r \neq 0$

11a) $\frac{a^2-25}{a^2+10a+25} : \frac{35-7a}{a^2+5a} = \frac{(a-5)(a+5)}{(a+5)^2} \cdot \frac{a(a+5)}{7(5-a)} = \frac{(a-5)\cdot a}{-7(a-5)} = -\frac{a}{7}$ $a \neq -5$, $a \neq 5$, $a \neq 0$

11f) $\frac{a(x^2-y^2)}{(x+y)^2} : \frac{a(x-y)^2}{3(x+y)} = \frac{a(x+y)(x-y)}{(x+y)^2} \cdot \frac{3(x+y)}{a(x-y)^2} = \frac{3}{(x-y)}$ $x \neq -y$, $x = y$, $a \neq 0$

V. SLOŽENÝ LV.

12a) $\frac{\frac{6u^2}{5v^3}}{\frac{4uv}{10}} = \frac{6u^2}{5v^3} : \frac{4uv}{10} = \frac{3}{6u^2} \cdot \frac{2}{5v^3} \cdot \frac{10}{4uv} = \frac{3u}{v^4}$ $v \neq 0$, $u \neq 0$

12f) $\frac{\frac{2u}{4u-6}}{\frac{6u^2}{2u-3}} = \frac{2u}{4u-6} : \frac{6u^2}{2u-3} = \frac{2u}{2(2u-3)} \cdot \frac{(2u-3)}{6u^2} = \frac{1}{6u}$ $u \neq \frac{3}{2}$, $u \neq 0$

13a) $\frac{\frac{x^2-100}{2x-1}}{\frac{2x+20}{4x^2-4x+1}} = \frac{x^2-100}{2x-1} : \frac{2x+20}{4x^2-4x+1} = \frac{(x-10)(x+10)}{2x-1} \cdot \frac{(2x-1)^2}{2(x+10)} = \frac{(x-10)(2x-1)}{2} =$

$= \frac{1}{2}(x^2-20x-x+10) = \frac{1}{2}(x^2-21x+10)$

$x \neq 0{,}5$, $x \neq 10$

b) $\frac{a^2+2ab+b^2}{\frac{x^2-25}{2x}} = \frac{a^2+2ab+b^2}{1} : \frac{x^2-25}{2x} = \frac{(a+b)^2}{1} \cdot \frac{2x}{x^2-25} = \frac{(a+b)^2 \cdot 2x}{(x^2-25)}$ (asi chyba vzadu!)

$x \neq 0$; $x \neq \pm 5$;

VI. PŘÍMAČKOVÉ PŘÍKLADY

14a) $\left(\frac{2t-1}{t+1} - \frac{2t+1}{t-1}\right) : \frac{t}{t-1} = \left(\frac{(2t-1)(t-1)-(2t+1)(t+1)}{(t+1)(t-1)}\right) \cdot \frac{(t-1)}{t} =$

$= \frac{2t^2-t-2t+1-(2t^2+3t+1)}{(t+1)} \cdot \frac{1}{t} = \frac{-5t-3t}{(t+1)} \cdot \frac{1}{t} = \frac{-6t}{(t+1)} \cdot \frac{1}{t} =$

$= -\frac{6}{(t+1)}$ $t \neq \pm 1$; $t \neq 0$

14b) $\frac{\frac{1+a}{1-a} - \frac{1-a}{1+a}}{\frac{1-a}{1+a} + \frac{1+a}{1-a}} = \frac{\frac{(1+a)^2-(1-a)^2}{(1-a)(1+a)}}{\frac{(1-a)^2+(1-a)^2}{(1+a)(1+a)}} = \frac{(1+a+1-a)(1+a-1+a)}{1-a^2} \cdot \frac{1-a^2}{1-2a+a^2+1+2a+a^2} =$

$\Rightarrow \frac{1 \cdot 2a}{2a^2+2} = \frac{a}{a^2+1}$ $a \neq \pm 1$

14c) $\left(\frac{u}{u+v} + \frac{v}{u-v}\right)\left(1 - \frac{2uv}{u^2+v^2}\right) = \left(\frac{u(u-v)+v(v+u)}{(u-v)(u+v)}\right)\left(\frac{u^2+v^2-2uv}{u^2+v^2}\right) = \frac{u^2-uv+v^2+uv}{u^2-v^2} \cdot \frac{(u-v)^2}{u^2+v^2} =$

$= \frac{u^2+v^2}{(u-v)(u+v)} \cdot \frac{(u-v)^2}{u^2+v^2} = \frac{u-v}{u+v}$ $u \neq \pm v$

Strana 2